MAT 407

Supplementary review problems - final exam

Work these problems in addition to working HW and previous exam problems. It will not be enough preparation for the exam to only do these problems.

1. Suppose X and Y have the following joint density function:

$$f(x,y) = \begin{cases} xe^{-(x+y)}, & x > 0, y > 0 \\ 0, & otherwise \end{cases}$$

Determine whether or not X and Y are independent and justify your answer.

2. a. Suppose X and Y are jointly distributed random variables. Show that in the general case, $Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$

b. If X and Y are independent, show that $Var(X+Y) = Var(X) + Var(Y)$.

3. Suppose X is an exponential r.v. with $\theta = 2$ and Y is exponential with $\theta = 3$. Also suppose X and Y are independent.
   a. Find the joint p.d.f. of X and Y.
   b. Find $P(X > Y)$.

4. Find the value of k that makes f(x) given below a p.d.f. for a random variable X.

$$f(x) = \begin{cases} \frac{k}{x^2}, & 2 \le x \le 5 \\ 0, & otherwise \end{cases}$$

5. Using your answer to part 4, find
   a. the expected value and variance of X.
   b. The cumulative distribution function of X, $F(x)$.
   c. P(X > 3.2).

6. In a student club of 24 members, 13 are in favor of a new campus policy, 7 are against it and the rest are neutral. A group of 4 students is to be selected at random from the club.
   a. How many possible committees of size 4 are there?
   b. If the committee is selected at random, what is the probability that two of the members on it are in favor of the policy and two against?
   c. Let X = number of members on the committee in favor of the policy and let Y = number against the policy. Find the joint probability mass function of X and Y.
   d. Find $P(X > Y)$.

7. A tire manufacturer knows that the mean life (in miles) of the company's tires is 50,000 with a standard deviation of 3500 miles. (this assumes usual wear and tear.) The lives of the tires follow a normal distribution.

a. What is the probability a tire will last beyond 60,000 miles, given that it already lasted 50,000?

b. What is the chances that a tire will wear out before 47000 miles?

c. What is the 75th percentile of the lives of the tires?

d. What is the 10th percentile for this distribution?

8. Suppose X is a discrete random variable with p.m.f. given by $P(X = x) = \frac{2^x}{3^{x+1}}$ for x = 0, 1, 2, ...

a. Verify that f is a legitimate p.m.f.

b. Find $P(X > 5)$

9. A coin is weighted so that $P(H) = 0.6$. Suppose it is flipped until heads comes up three times. Let X = the number of flips required to obtain three heads.

a. What is the probability distribution of X?

b. What is the probabiltiy that it will take more than 10 flips to obtain 3 heads?

c. How many flips do you expect it to take to obtain the three heads?