

MAT 407

Hand-in assignment #8

Due on or before Tuesday, December 13

You may work with a classmate, but you must write up your own final solutions and also indicate with whom you worked. Please staple any multiple pages together.

1. Suppose X and Y are jointly discrete with probability mass function given by

$$f(x,y) = \frac{x+y}{15}, \quad x = 0, 1 \quad y = 1, 2, 3$$

- Graph the support of the function.
- Find $P(X < Y)$.
- Find $P(X = 0, Y \geq 2)$.
- Find the marginal distributions, $f_1(x)$ and $f_2(y)$. Are X and Y independent? Why or why not?
- Find μ_x, μ_y, σ_x and σ_y using the marginal distributions.
- Find $Cov(X, Y)$ and the correlation coefficient. What does the correlation coefficient tell you about the relationship between X and Y ?

2. Suppose X and Y are jointly continuous with the following joint probability density function:

$$f(x,y) = \begin{cases} k(x-y) & 0 \leq y \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

- Sketch the support of the function.
- Find the value of k .
- Find $P(X > \frac{1}{2}, Y < \frac{1}{4})$.
- Find the marginal distributions $f_1(x)$ and $f_2(y)$. Are X and Y independent? Why or why not?
- Find μ_x, μ_y, σ_x and σ_y using the marginal distributions.
- Find $Cov(X, Y)$ and the correlation coefficient. What does the correlation coefficient tell you about the relationship between X and Y ?

3. Suppose X and Y are identically distributed random variables (they are not necessarily independent.)

Prove that $Cov(X+Y, X-Y) = 0$. Hint: the fact that X and Y have the same distributions means that X and Y have equal means and equal variances.