Selected solutions to review problems for Exam 2:

1. a. $X \sim B\left(10, \frac{1}{4}\right)$

b. $\mu = np = 2.5$

c. $P(X = 0) = \left(\frac{3}{4}\right)^{10} = 0.056$

d. $P(X \geq 6) = 1 - P(X \leq 5) = 1 - F(5) = 1 - 0.98 = 0.02$ (using the table or the TI-83 to find F(5))

2. If X is uniform on [1,5], then

$$f(x) = \begin{cases} 1/4, & 1 \leq x \leq 5 \\ 0, & otherwise \end{cases} \quad \text{and} \quad F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{4}(x-1), & 1 \leq x \leq 5 \\ 1, & x > 5 \end{cases}$$

c. $\mu = 3, \quad \sigma^2 = 4/3$

d. $P(X > 3) = 1/2 \quad P(X < 0) = 0 \quad P(1 < X < 3) = 1/2$

3. a. X is Poisson with rate $\lambda = 4$ per week, because X is counting something that occurs at random, relatively infrequently.

b. $P(X = 0) = \frac{e^{-4}4^0}{0!} = e^{-4} = 0.0183$

c. $P(X \leq 5) = F(5) = 0.785$ (using table or TI-83)

d. $P(X > 6) = 1 - P(X \leq 6) = 1 - F(6) = 0.111$

4. a. W is exponential with $\theta = 1/4$ (of a week), because $\lambda = 4$ calls per week.

b. $P(W > 3/7) = 1 - F(3/7) = 1 - \left(1 - e^{-4 \cdot \frac{3}{7}}\right) = 0.18$

Here, we were using the cdf of the exponential. You could also directly integrate $\int_{3/7}^{\infty} 4e^{-4x}dx$.

c. $E(W) = 1/4$ of a week. That is, on average you would expect to wait 1/4 of a week, or 1.75 days for the first call.

$Var(W) = (1/4)^2 = 1/16$

5. Note that Y is a geometric r.v. with p = 0.6. We spent some time in class and on the homework showing a couple of things, one of which was that for a geometric r.v., $P(Y > k) = q^k$, where $q = 1 - p$. We also showed that the geometric r.v. has the "memoryless" property, which states that

$$P(Y > s + t \mid Y > s) = P(Y > t)$$

For this problem, that means $P(Y > 10 \mid Y > 7) = P(Y > 3) = (0.4)^3 = 0.064$ (i.e. take s = 7 and t = 3).