

## MAT 407

### Useful facts

1. Sum of a geometric series:  $\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$ , for  $|r| < 1$

2. Finite geometric sum:  $\sum_{k=0}^n ar^k = \frac{a(1-r^{n+1})}{1-r}$

3. Binomial Theorem:  $(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}$

4. For any real number  $a$ ,  $\sum_{k=0}^{\infty} \frac{a^k}{k!} = e^a$ .

### Practice problems

1. Let  $X$  be a Bernoulli random variable with parameter  $p$ . Find  $M(t)$  and  $M'(0)$ . What is the mean and variance of  $X$ ?

2. Use the Binomial Theorem to expand  $(3r - 2s)^5$

3. Let  $Y$  be Binomial  $(n, p)$ . Find  $M(t)$  and use it to prove that  $E(Y) = np$  and that  $Var(Y) = npq$ . Hint: use the Binomial Theorem and the fact that  $e^{tx} = (e^t)^x$ .

4. Suppose  $X$  is a discrete uniform random variable on  $\{1, 2, 3, 4, 5\}$ . Recall that this means that  $f(x) = \frac{1}{5}$  for  $x = 1, \dots, 5$ . Find  $E(X)$ ,  $Var(X)$  and  $M(t)$ .

5. Suppose  $X$  is a Poisson random variable with parameter  $\lambda$ . Then  $f(x) = \frac{e^{-\lambda} \lambda^x}{x!}$ .

a. Show that  $\sum_{x=0}^{\infty} f(x) = 1$ .

b. Find  $M(t)$ .

c. What are  $E(X)$  and  $Var(X)$ ?