1. For \( n \in \mathbb{Z}_+ \), prove that
\[
\sum_{i=1}^{n} \frac{1}{(3i - 2)(3i + 1)} = \frac{n}{3n+1}.
\]

2. For \( n \in \mathbb{Z}_+ \), find and prove a formula for
\[
\sum_{i=1}^{n} (2i - 1).
\]

3. For \( n \in \mathbb{Z}_+ \), find and prove a formula for
\[
\sum_{i=1}^{n} \frac{1}{i(i + 1)}.
\]

4. For \( n \in \mathbb{Z}_+ \), prove that
\[
\sum_{i=1}^{n} (2i - 1)^2 = \frac{n(2n-1)(2n+1)}{3}.
\]

5. Let \( q \) be a real number other than 1. Use induction on \( n \in \mathbb{Z}_+ \) to prove that
\[
\sum_{i=0}^{n-1} q^i = \frac{q^n - 1}{q - 1}.
\]

6. Prove that \( A \times (B \setminus C) = (A \times B) \setminus (A \times C) \).

7. Let \( f : X \to Y \) be a function, and let \( A \) be a subset of \( X \) and \( B \) be a subset of \( Y \). Prove that
\[
A \subseteq f^{-1}(f(A))
\]
and
\[
f(f^{-1}(B)) \subseteq B.
\]

8. Let \( I \) be a nonempty index set, let \( \mathcal{A} = \{A_i \mid i \in I\} \) be an indexed family of sets, and let \( B \) be a set. Prove that
\[
\left( \bigcup_{i \in I} A_i \right) \setminus B = \bigcup_{i \in I} (A_i \setminus B).
\]
9. Define a bijective function $f : ]0, 1[ \to \mathbb{R}$. Justify your claims.

10. Define a bijective function $f : [0, 1] \to ]0, 1[$. Justify your claims.

11. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = e^{-x^2}$. Is $f$ invertible? Justify your conclusion.

12. Given $f : A \to B$ and $g : B \to C$, let $h = g \circ f$. Determine which of the following statements are true. Give proofs for the true statements and counterexamples for the false statements.

   (a) If $h$ is injective, then $f$ is injective.
   (b) If $h$ is injective, then $g$ is injective.
   (c) If $h$ is surjective, then $f$ is surjective.
   (d) If $h$ is surjective, then $g$ is surjective.

13. Consider $f : A \to B$ and $g : B \to A$. Answer each question below by providing a proof or a counterexample.

   (a) If $f(g(y)) = y$ for all $y \in B$, does it follow that $f$ is a bijection?
   (b) If $g(f(x)) = x$ for all $x \in A$, does it follow that $f(g(y)) = y$ for all $y \in B$?

14. Let $A$ be the set of subsets of $\{1, 2, \ldots, n\}$ that have even size, and let $B$ be the set of subsets of $\{1, 2, \ldots, n\}$ that have odd size. Establish a bijection from $A$ to $B$. (⋆)