1. Prove that
\[\bigcup_{i \in I} \bigcap_{j \in J} A_{i,j} = \bigcap_{f \in \prod_{i \in I} J_i} \bigcup_{i \in I} A_{i,f(i)}\]
and
\[\bigcap_{i \in I} \bigcup_{j \in J} A_{i,j} = \bigcup_{f \in \prod_{i \in I} J_i} \bigcap_{i \in I} A_{i,f(i)}\]

2. Prove that
\[\prod_{i \in I} \left( \bigcup_{j \in J_i} A_{i,j} \right) = \bigcup_{f \in \prod_{i \in I} J_i} \left( \prod_{i \in I} A_{i,f(i)} \right)\]
and
\[\prod_{i \in I} \left( \bigcap_{j \in J_i} A_{i,j} \right) = \bigcap_{f \in \prod_{i \in I} J_i} \left( \prod_{i \in I} A_{i,f(i)} \right)\]

3. If \( A_0, A_1, \ldots \) is a sequence of sets, then there are pairwise disjoint sets \( B_i \subseteq A_i \) such that \( \bigcup_{n \in \mathbb{N}} B_i = \bigcup_{i \in \mathbb{N}} A_i \).

4. Define
\[\liminf_{n \to \infty} A_n := \bigcup_{n=1}^{\infty} \bigcap_{m=n}^{\infty} A_m,\]
and
\[\limsup_{n \to \infty} A_n := \bigcap_{n=1}^{\infty} \bigcup_{m=n}^{\infty} A_m,\]
and we say that the sequence \( \{A_m\} \) is convergent if these two sets are the same set, say \( A \), in which case we say that the \textit{limit} of the sets \( \{A_m\} \) is \( A \). Then
(a) \( \liminf_n A_n \subseteq \limsup_n A_n \),
(b) \( \liminf_n A_n \) consists of those elements that belong to all but finitely many of the \( A_n \)’s.
(c) \( \limsup_n A_n \) consists of those elements that belong to infinitely many of the \( A_n \)’s.
5. Let \( X \) be a set and for a subset \( A \) of \( X \) consider its characteristic function

\[
\chi_A(x) = \begin{cases} 
1 & \text{if } x \in A, \\
0 & \text{if } x \notin A.
\end{cases}
\]

Let \( \mathcal{P}(X) \) denote the power set of \( X \). Let \( f : \mathcal{P}(X) \to \{0, 1\}^X \) be defined by

\[
f(A) = \chi_A.
\]

Prove that \( f \) is a bijection.