Salem State College
Mathematics Department
Autumn, 2007

Course: MAT 420: Special Problems Seminar
Section: 01
Course webpage: http://salemstate.edu/~lpoitevin/Fall-2007/proofs.html
Room: SB 311
Schedule: MWF 10:00 - 10:50
Instructor: L. Pedro Poitevin, Assistant Professor
Office: Sullivan Building 308B
Office hours: TuTh 4:00 - 5:00; WF: 11:00 - 12:30
Office phone number: (978) 542-6995
E-mail: lpoitevin@salemstate.edu

Course description: An introduction to mathematical proof techniques and the fundamental notions of higher mathematics. Topics will include the basics of propositional logic, set theory, mathematical induction, functions, equivalence relations, and cardinality with an emphasis on writing proofs.

Course goals:

1. To provide students with a good understanding of what constitutes a well-written proof.
2. To provide students with the required skills to understand mathematical proofs in higher mathematics.
3. To provide students with the ability to write elementary mathematical proofs.

Learning objectives: A student who passes this course should be able to:

1. Identify mathematical statements.
2. Read and understand mathematical definitions.
3. Identify the converse and the contrapositive of an implication.
4. Understand the meaning of bi-implications and how to prove them.
5. Identify and prove universal and existential statements.

6. Unravel the meaning of statements involving several negations; in particular, understand how to transform statements of type \( \neg \forall x P(x) \) into statements of type \( \exists x \neg P(x) \), and likewise with existential statements.

7. Unravel the meaning of statements with nested quantifiers, and understand how to prove them.

8. Understand the law of the excluded middle, and apply it to prove mathematical statements by contradiction.

9. Apply the technique of proof by mathematical induction whenever appropriate.

10. Understand how to prove uniqueness results.

11. Correctly write \( \epsilon—\delta \) proofs and their relatives.

**Mathematical Content:** The emphasis of the course will be on proof techniques. As we go along, we will touch on certain mathematical topics to accompany our discussions of proof techniques. These topics will include predicate logic, set theory, functions, equivalence relations, partial orderings, cardinality, elementary arithmetic, the structure of the real numbers (including infima and suprema of sets), and fundamentals of calculus in one real variable.

**Textbook:** *How to Read and Do Proofs*, by Daniel Solow (Required).

**Class format:** These lectures will follow a hybrid approach: a fraction of class time will be devoted to lectures, and the rest to class work. Lectures will help you understand some of the finer points of the subject matter. Class work will allow you to gauge your understanding of the subject matter in the company of peers who will help you through the learning process. It will also give you an opportunity to explain to others what you have learned, which is a very effective way of solidifying one’s understanding in general. During class work, I will oversee your work and provide helpful hints. As a further element of your active participation in this class, you will often be asked to spend a minute responding to a short feedback question at the end of the lecture. I will listen and respond to this feedback.
Exams: There will be four exams: a take-home and an in-class midterm, as well as a take-home and an in-class final. The take-home midterm and final will count for 15% of the course grade each. The in-class midterm and final will count for 10% of the course grade each. The in-class exams are scheduled as follows:

<table>
<thead>
<tr>
<th>Exam</th>
<th>Date/Time</th>
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<tbody>
<tr>
<td>Midterm</td>
<td>Tentatively Friday, October 12 10:00 - 10:50</td>
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<tr>
<td>Final</td>
<td>Thursday, December 13 8:00 - 10:00</td>
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Final project: Each individual student will be responsible for a final project. The successful completion of such a project will entail producing a well-written, professional-looking exposition of a mathematical topic assigned to the student, as well as a short presentation on aspects of the topic. Along the semester there will be scheduled meetings with students to provide feedback and guidance on the final project. Students should expect the project to undergo several revisions, and so it is advisable to start working rather early in the semester. The final project will account for 20% of the student’s final grade.

Written assignments: Homework in this class will consist of assignments that will require students to write very careful and detailed proofs and solutions. After all, the purpose of this course is to help students develop the skills necessary (and hopefully sufficient) for writing mathematically correct proofs. Written assignments will be collected by a strict deadline and graded and returned to students expeditiously. A student who submits a written assignment by the corresponding deadline will be entitled to a second submission, once the assignment is graded and returned. Written assignments will account for 20% of the student’s grade.

In-class presentations: Students will be responsible for occasionally presenting their work in class. In-class presentations account for 10% of the student’s final grade.

Grading scheme: I reserve the right to change the following grading scheme, but it will very likely stand:
Written assignments 20 %
In-class presentations 10 %
In-class midterm 10 %
Take-home midterm 15 %
Final project 20 %
In-class final exam 10 %
Take-home final exam 15 %

Attendance policy: An *advance notice* for an absence to class is typically an e-mail sent to me 12 hours or more in advance of the class meeting the student will not attend. *Permission for absence* is typically an e-mail from me to the student to acknowledge receipt of an advance notice. *Excused absences* are absences for which I have advance notice, and for which the student has a permission for absence. Excused absences, if not excessive in number, will not negatively affect a student’s grade. An unexcused absence during a day when an assignment, quiz, or exam is due will result in a grade of zero for the assignment, quiz, or exam. Initially, I will not take attendance, but this may change if desirable. I reserve the right to penalize students with more than two unexcused absences by reducing their final grade by one letter grade or more. In the event that I wish to exercise this right, notice will be given to students in advance. **The student is responsible for completing all course requirements and for keeping up with all that goes on in the course (whether or not the student is in attendance).**

Statement on Equality of Access: Salem State College is committed to providing equal access to educational experience for all students in compliance with Section 504 of The Rehabilitation Act and The Americans with Disabilities Act and to providing all reasonable academic accommodations, aids and adjustments. Any student who has a documented disability should speak with the instructor immediately. Students with disabilities who have not previously done so should provide documentation to and schedule an appointment with the Office for Students with Disabilities and obtain appropriate services.