1. Let $A = \{a, b\}$ and let $R = \{(a, b)\}$. Is $R$ an equivalence relation on $A$? Justify your conclusion.

2. Let $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} | |x| + |y| = 4\}$. Then $R$ is a relation on $\mathbb{R}$. Is $R$ an equivalence relation on $\mathbb{R}$? Justify your conclusion.

3. A relation $R$ is defined on $\mathbb{Z}$ as follows: for all $a, b \in \mathbb{Z}$, $aRb$ if and only if $|a - b| \leq 3$. Is $R$ an equivalence relation on $\mathbb{R}$? Justify your conclusion.

4. Let $A = \{1, 2, 3, 4, 5\}$. The identity relation on $A$ is $I_A = \{(1,1), (2,2), (3,3), (4,4), (5,5)\}$. Find an equivalence relation on $A$ that is different from $I_A$.

5. Let $A = \{a, b, c\}$. For each of the following, draw a directed graph that represents a relation with the specified properties:
   (a) A relation on $A$ that is symmetric but not transitive.
   (b) A relation on $A$ that is transitive but not symmetric.
   (c) A relation on $A$ that is symmetric and transitive but not reflexive.
   (d) A relation on $A$ that is not reflexive, is not symmetric, and is not transitive.
   (e) A relation on $A$, other than the identity relation, that is an equivalence relation.

6. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 - 4$ for each $x \in \mathbb{R}$. Define a relation $\sim$ on $\mathbb{R}$ as follows: for $a, b \in \mathbb{R}$, $a \sim b$ if and only if $f(a) = f(b)$.
   (a) Is the relation $\sim$ an equivalence relation on $\mathbb{R}$? Justify your conclusion.
   (b) Determine all real numbers in the set $C = \{x \in \mathbb{R} | x \sim \sqrt{2}\}$.

7. Let $\sim$ and $\approx$ be relations on $\mathbb{Z}$ defined as follows:
   • For $a, b \in \mathbb{Z}$, $a \sim b$ if and only if 2 divides $a + b$.
   • For $a, b \in \mathbb{Z}$, $a \approx b$ if and only if 3 divides $a + b$.
   (a) Is $\sim$ an equivalence relation on $\mathbb{Z}$? Justify your conclusion.
   (b) Is $\approx$ an equivalence relation on $\mathbb{Z}$? Justify your conclusion.

8. Define the relation $\sim$ on $\mathbb{R}$ as follows: for $x, y \in \mathbb{R}$, $x \sim y$ if and only if $x - y \in \mathbb{Q}$.
   (a) Prove that $\sim$ is an equivalence relation on $\mathbb{R}$.
   (b) List four different real numbers that are in the equivalence class of $\sqrt{2}$. 


(c) If $a \in \mathbb{Q}$, what is the equivalence class of $a$?
(d) Prove that $[\sqrt{2}] = \{r + \sqrt{2} \mid r \in \mathbb{Q}\}$.
(e) If $a \in \mathbb{Q}$, prove that there is a bijection from $[a]$ to $[\sqrt{2}]$.

9. Let $U$ be a fixed infinite universe, over which all sets from now on invoked will range. Let $\leq$ be the following relation on the power set $2^U$ of $U$: for $A, B \subseteq U$, $A \leq B$ if and only if there exists a function $f : A \rightarrow B$ such that $f$ is injective.

(a) Is $\leq$ a reflexive relation on $2^U$? Justify your answer.
(b) Is $\leq$ a symmetric relation on $2^U$? Justify your answer.
(c) Is $\leq$ a transitive relation on $2^U$? Justify your answer. (Hint: you may here appeal to any relevant exercise from previous worksheets.)
(d) Is $\leq$ an antisymmetric relation on $2^U$? Justify your answer. (**) 
(e) Is $\leq$ a complete relation on $2^U$? Justify your answer. (**)