(1) Define an explicit bijection between $\mathbb{N}$ and $\mathbb{Z}_+$, and prove that it is indeed a bijection.

(2) Define an explicit bijection between $\mathbb{N}$ and $\mathbb{Z}$, and prove that it is indeed a bijection.

(3) Define an explicit bijection between $\mathbb{N}$ and $\mathbb{Q}_+$.

**Definition.** A set $A$ is said to be *countable* if there exists a bijection between $A$ and $\mathbb{N}$. (For example, $\mathbb{Q}$ is countable.)

(4) Prove that $]0, 1[$ is not countable.

(5) Since we have shown $]0, 1[$ to be equinumerous with $\mathbb{R}$, make the appropriate conclusion about $\mathbb{R}$.

   (Hint: time to go back to your solution of the appropriate problem from a past worksheet.)

(6) Suppose that $I$ is a countable set, and that for each $i \in I$, $A_i$ is a countable set. Prove that $\bigcup_{i \in I} A_i$ is a countable set.

(7) Let $A$ be the set of all functions $f : \mathbb{N} \to \mathbb{Q}$ satisfying the following property: there exists $n \in \mathbb{N}$ such that for all $p \in \mathbb{N}$, $n \leq p \Rightarrow f(n) = f(p)$. Prove that $A$ is countable.