1. Compute
\[ \lim_{n \to \infty} \left| \sin \left( \pi \sqrt{n^2 + n + 1} \right) \right|. \]

2. Let \( k \) be a positive integer and \( \mu \) a positive real number. Prove that
\[ \lim_{n \to \infty} (n^k) \left( \frac{\mu}{n} \right)^k \left( 1 - \frac{\mu}{n} \right)^{n-k} = \frac{\mu^k}{e^\mu \cdot k!}. \]

3. For \( a \in \mathbb{R} \), calculate
\[ \lim_{n \to \infty} \frac{1}{n} \left( \left( a + \frac{1}{n} \right)^2 + \left( a + \frac{2}{n} \right)^2 + \cdots + \left( a + \frac{n-1}{n} \right)^2 \right). \]

4. Find all cluster points of the sequence \((a_n)\) described by
\[ a_n = \left( 1 + \frac{1}{n} \right)^n (-1)^n + \sin \left( \frac{n\pi}{4} \right) \]
and find \( \lim \sup a_n \) and \( \lim \inf a_n \).

We say that a series \( \sum x_i \) in a normed space \((X, \| \cdot \|)\) is absolutely convergent if \( \sum \|x_i\| < \infty \). Recall that \( \sum x_i \) is called convergent if the sequence \( s_n = \sum_{i=1}^{n} x_i \) is convergent in \( X \).

5. Show that a normed space \((X, \| \cdot \|)\) is complete if and only if every absolutely convergent series in \( X \) is convergent.

6. Does there exist a positive sequence \((a_n)\) such that both \( \sum a_n \) and \( \sum 1/(n^2 a_n) \) are convergent?