1. Describe the atomic truth assignments on \( \{a_1, \ldots, a_n\} \) that satisfy the proposition 
\[ p = ((a_1 \rightarrow a_2) \land (a_2 \rightarrow a_3) \land \cdots \land (a_{n-1} \rightarrow a_n)) \] (15 points)
2. Let $L$ be a language and let $A[x, y]$ be an arbitrary $L$-formula with two free variables. Is the sentence $\forall x \exists y A[x, y] \rightarrow \exists y \forall x A[x, y]$ satisfied in any $L$-structure? Explain. (15 points)

3. Let $L = \{R\}$, where $R$ is a binary relation symbol. Let $\mathcal{A} = (\mathbb{Q}, \leq)$ and $\mathcal{B} = (\mathbb{Z}, \leq)$. Write down an $L$-sentence which is true in $\mathcal{A}$ but false in $\mathcal{B}$. (15 points)
4. The language $L$ consists of a single binary relation symbol $R$. Consider the $L$-structure $\mathcal{A}$ whose underlying set is $A = \{ n \in \mathbb{N} \mid n \geq 2 \}$ and in which $R$ is interpreted by the relation ‘divides’, i.e. $R^\mathcal{A}$ is defined for all integers $m, n \geq 2$ by the condition: $(m, n) \in R^\mathcal{A}$ if and only if $m$ divides $n$. Describe the set of elements of $A$ that satisfy the $L$-formula $\varphi = \forall y \forall z ((R(y, x) \land R(z, x)) \rightarrow (R(y, z) \lor R(z, y)))$. (15 points)

5. Show that the following sets are 0-definable in the corresponding structures:

(a) The ordering relation $\{(m, n) \in \mathbb{N}^2 \mid m < n \}$ in $(\mathbb{N}, 0, +)$. (15 points)
(b) The set of prime numbers in the semiring $\mathcal{N} = (\mathbb{N}, 0, 1, +, \cdot)$. (15 points)

(c) The set $\{2^n \mid n \in \mathbb{N}\}$ in the semiring $\mathcal{N}$. (15 points)
(d) The set \( \{ a \in \mathbb{R} \mid f \text{ is continuous at } a \} \) in \((\mathbb{R}, <, f)\) where \( f : \mathbb{R} \to \mathbb{R} \) is any function. (15 points)

6. The language \( L \) consists of a single binary relation symbol \( E \). Write down sentences expressing that the interpretation of \( E \) is an equivalence relation having infinitely many equivalence classes, each of which has infinitely many elements. (20 points)