1. Define $d : \mathbb{R} \times \mathbb{R} \to [0, \infty)$ by $d(x, y) = |x - y|$. Show that $(\mathbb{R}, d)$ is a metric space.

2. Let $d : \mathbb{R}^2 \times \mathbb{R}^2 \to [0, \infty)$ be defined by $d_2((x_1, y_1), (x_2, y_2)) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. Show that $(\mathbb{R}^2, d_2)$ is a metric space.

3. Let $X$ be a set and let $d : X \times X \to [0, \infty)$ satisfy the following two properties:
   (a) For all $x, y \in X$, $d(x, y) = 0$ if and only if $x = y$.
   (b) For all $x, y, z \in X$, $d(x, y) \leq d(z, x) + d(z, y)$.

   Show that $(X, d)$ is a metric space.

4. Prove that any open ball is an open set.

5. Prove that any closed ball is a closed set.

6. If $x_0$ is an accumulation point of a set $A$ in a metric space $(X, d)$, show that for any $r > 0$, the open ball $B(x_0, r)$ contains infinitely many points of $A$.

7. Prove or give a counterexample: “The closure $\overline{B(x_0, r)}$ of an open ball $B(x_0, r)$ in a metric space is equal to the closed ball $B[x_0, r]$.”

8. Let $(X, d)$ be a metric space, and let $A, B$ be subsets of $X$. Show that $A \subseteq \overline{A}$, $\overline{A} = \overline{A}$, $\overline{A \cup B} = \overline{A} \cup \overline{B}$, and $\overline{A \cap B} \subseteq \overline{A} \cap \overline{B}$.

9. Show that every Cauchy sequence is bounded.