Marks

Students may use calculators and one page of handwritten notes.

1. Assume the marginal cost, $M$, is such that $M'(x) > 0$ and $M''(x) < 0$ for all $x$, where $x$ is the number of pounds of flour made, and the following information is known:

<table>
<thead>
<tr>
<th>Weight of flour, $x$</th>
<th>Marginal cost, $M$</th>
<th>Total cost, $C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 pounds</td>
<td>$2.00 per pound</td>
<td>$300</td>
</tr>
<tr>
<td>120 pounds</td>
<td>$2.30 per pound</td>
<td>?</td>
</tr>
<tr>
<td>140 pounds</td>
<td>$2.50 per pound</td>
<td>??</td>
</tr>
</tbody>
</table>

(a) Arrange in ascending order (i.e. smallest first and largest last) the numerical values of the following four quantities:

$$\frac{M(140) - M(120)}{20}, \quad \frac{M(120) - M(100)}{20}, \quad M'(120), \quad 0.$$

Either calculate the following exactly if possible, or give the answer in the form $a \pm b$, where $a$ and $b$ are values you need to specify.

(b) Find $M'(120)$ (the instantaneous rate of change of $M$ with respect to $x$ when $x = 120$).

(c) Find $C(140)$ (the total cost when 140 pounds of flour are made).

2. Find the derivative, $\frac{dy}{dx}$, if

(a) $y = 2x^5 - \sqrt{x} + \frac{1}{x}$

(b) $y = x^4 e^x$

(c) $y = 12e^{-0.2x^3}$

(d) $y = \frac{3\ln(x)}{x^5 + 3}$

(e) $y = \int_4^x 8te^{-t^2} \, dt$

3. Assume a gas station pumps 900 gallons of gas per month at a price of $1.60 per gallon. The amount of gas it pumps decreases at a rate of 20 gallons per month and the price increases at a rate of 20 cents per gallon per month. Find the instantaneous rate at which the monthly revenue is increasing per month.

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4. Evaluate the following limits:
   (a) \( \lim_{z \to 2} \frac{z^2 - 5z + 6}{z - 2} \)
   (b) \( \lim_{x \to \infty} \frac{24x^3 + 5x^2}{2x^4 - 4x^2} \).

5. Find the following integrals:
   (a) \( \int_5^{10} (4 + 8x) \, dx \)
   (b) \( \int (x^3 + 2)e^{x^4 + 8x} \, dx \)
   (c) \( \int_{0.001}^{900} \frac{8}{x} \, dx \)
   (d) \( \int_2^5 \left[ \frac{d}{dx}(x^3 \ln x) \right] \, dx \)

6. Assume the revenue per year (or sales rate), \( R \), of a company is 5 million dollars per year on January 1, 2001 and is undergoing exponential growth with a growth constant of 40% per year; that is \( \frac{dR}{dt} = 0.40R \), where \( t \) is the number of years after January 1, 2001.
   (a) Find the sales rate on January 1, 2005.
   (b) Find when the sales rate will be 20 million dollars per year.
   (c) Find the total accumulated sales between January 1, 2001 and January 1, 2005.

7. Consider the function \( f(x) = \frac{x^3 + 1}{x} \). Assume the first and second derivatives of this function are as follows: \( f'(x) = \frac{2x^3 - 1}{x^2} \) and \( f''(x) = \frac{2x^3 + 2}{x^3} \). Use the derivatives to discuss and sketch the graph of this function. (Identify and label all relative maxima and minima and inflection points. Find intervals where the curve is concave upward or downward and show the concavity clearly. Also, find the behavior of the graph for \( x \) large and for \( x \) near any discontinuities of the function, and identify and label all intercepts and asymptotes.)

8. Sketch the region \( R \) enclosed between the graphs of \( y = e^x \), \( y = 8 - e^x \), and \( x = 2 \), and find its area.

9. A rectangular billboard is to be made with 900 square feet of printed area and with margins of 6 feet at the top and bottom and 12 feet on each side. Use calculus to find the dimensions of the printed area of the billboard if its total area is to be minimized.